Discrete Mathematics and Computer Science Seminars
Department of Mathematical Sciences, Sharif University of Technology

The number of subsets of integers with no k-term arithmetic progression

Hong Liu
Mathematics Institute,
University of Warwick, Coventry, UK
h.liu.9@warwick.ac.uk

Hong Liu is a postdoc at the University of Warwick working with Oleg Pikhurko. He completed his Ph.D. at the University of Illinois at Urbana Champaign in December 2015, advised by József Balogh.

Abstract

Addressing a question of Cameron and Erdős, we show that, for infinitely many values of n, the number of subsets of $\{1, 2, \ldots, n\}$ that do not contain a k-term arithmetic progression is at most $2^{O(r_k(n))}$, where $r_k(n)$ is the maximum cardinality of a subset of $\{1, 2, \ldots, n\}$ without a k-term arithmetic progression. This bound is optimal up to a constant factor in the exponent. For all values of n, we prove a weaker bound, which is nevertheless sufficient to transfer the current best upper bound on $r_k(n)$ to the sparse random setting. To achieve these bounds, we establish a new supersaturation result, which roughly states that sets of size $\Theta(r_k(n))$ contain superlinearly many k-term arithmetic progressions.

For integers r and k, Erdős asked whether there is a set of integers S with no $(k+1)$-term arithmetic progression, but such that any r-coloring of S yields a monochromatic k-term arithmetic progression. Nešetřil and Rödl, and independently Spencer, answered this question affirmatively. We show the following density version: for every $k \geq 3$ and $\delta > 0$, there exists a reasonably dense subset of primes S with no $(k+1)$-term arithmetic progression, yet every $U \subseteq S$ of size $|U| \geq \delta|S|$ contains a k-term arithmetic progression.

(This is joint work with József Balogh and Maryam Sharifzadeh.)

Wednesday, **7 Mehr 1395** (28 September 2016), **12:45-14:00**
Room **317,** Department of Mathematical Sciences

http://mehr.sharif.ir/ combinatorics
combinatorics@mehr.sharif.ir